Choice of new attributes in the 'Elimination by Aspects' duopoly

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Abstract

The "Elimination by aspects" (EBA) duopoly of product differentiation (Laurent, 2006a) was constructed from the discrete model of probabilistic choice worked out by Tversky (1972a,b). In this framework, an unique price equilibrium exists with a "differentiation by attributes", which embodies horizontal and vertical differentiations as possible special cases. This paper extends this analysis by studying a two-stage game in which firms choose the specific attributes of their product (innovation) and then compete in prices. At the price equilibrium, the "competitive effect", present in pure vertical differentiation models, is replaced by a "differentiation effect" in this EBA duopoly. Subgame perfect Nash equilibria are shown to exist with exogenous costs but also with attributes-dependent unit and fixed costs. At the equilibrium, products are generally differentiated both horizontally and vertically. But a purely vertical outcome may also occur when costs of innovation are strongly convex or when consumers are very sensible to the price levels. When costs are endogenous, the social optimum is achieved for a pure horizontal differentiation. Thus, there is too much differentiation at the equilibrium : the vertical dimension induces a strong raise of prices, which also reduces the welfare.

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1 Introduction

Economic models usually suppose that the products are differentiated either according to a vertical dimension, the level of quality, or according to a horizontal dimension, the type of variety. However, most markets possess simultaneously the two previous dimensions and it seems not plausible that the economic agents (consumers or firms) use such an abstract (and "academic") classification to make their decisions.

The observation rather shows that the consumers choose a good according to the *attributes* it possesses or not: for example, the presence of a particular safety accessory can be a decisive factor for the purchase of a car. In the companies, product managers have perceived long ago the necessity to develop some *specific attributes* for their products, in order to obtain a comparative advantage. Thus, the decision problem of a marketing service may be better described by a choice among a set of product attributes rather than by a choice of "varieties" or "qualities". A firm can choose to add to its product an attribute already existing on the market in order to cancel the competing advantage of a rival (imitation), or it can propose a new attribute, in order to gain a competing advantage (innovation). However, proposing new attributes increases fixed R & D costs or unit costs of production. This article is devoted to the description of the choice of innovative attributes by the firms and to the analysis of the resulting situation in terms of welfare.

The choice of qualities with endogenous (quality-dependent) costs has been largely studied in the economic literature, both in the deterministic pure vertical differentiation (or "PVD") models (Ronnen, 1991, Motta, 1993) and in the logit discrete choice model (Anderson, De Palma and Thisse, 1992). But the "elimination by aspects" (or "EBA") model, initially proposed by Tversky (1972a, b) is the first to allow the study of the attributes choices. Indeed, the products are described by their set of attributes[1] and these attributes are at the center of the decision-making process of the agents. Tversky suggests describing individual choices as the result of a stochastic process involving a successive elimination of the products :

(a) the attributes common to all goods are eliminated because they can not allow to discriminate between goods during the choice process ;

(b) an attribute is randomly selected and all the products that do not have it are eliminated. The higher the utility of a characteristic is, the larger the probability of selecting this characteristic is ;

(c) if the remaining goods still have specific attributes, one goes back to the first stage. On the contrary, if all goods have the same attributes, the procedure ends. If only one product remains, it is selected by the consumer. Otherwise, all the remaining goods have the same probability to be selected.

This discrete choices model allows to take into account an interesting form of the imperfect rationality of consumers choosing among differentiated products. The probabilities of the EBA model are used by Laurent (2006a) to construct the demands of a duopoly. In such a structure, there is an unique price Nash equilibrium if the unit cost of the firm having the highest level of attributes is sufficiently high compared to that of its rival. When this equilibrium exists, the model provides a general framework of *differentiation by the specific attributes* which integrates the classical forms of differentiation. Thus, when the specific attributes of the two goods provide the same positive level of utility to the consumers, the differentiation is horizontal. When a single product has all the specific attributes available on the market, and the other

[1] Tversky employs the term of "aspects": for our part, we indifferently use "attributes" or "characteristic"

none, differentiation is vertical. Finally, when each good has some specific attributes but that provide different utility levels to the consumers, the two previous dimensions of differentiation are simultaneously taken into account. Note that the number of characteristics implying the product differentiation is chosen endogenously by firms in the EBA model whereas it is fixed a priori in existing models with multiple differentiations (see for instance Neven and Thisse, 1990, or Irmen and Thisse, 1998).

This article extends the previous analysis by studying a two-stage game in which firms choose the specific new attributes of their product and then compete in prices. We initially suppose that the addition of the specific attributes does not generate any additional cost. A comparative statics analysis of the EBA profits reveals that the convergence from the lowest level of attributes to the highest increases the differentiation on the market. Conversely, in PVD models, such a convergence of qualities decreases the differentiation. This observation opens perspectives in the field of economics of innovation. Indeed, the two dimensions of differentiation could now be taken into account in dynamic models, only by changing an assumption on the evolution of profits.

Then, when the costs are not linked with the attributes, firms choose the highest possible levels of attributes, which is consistent with the intuition and the results of the logit model (Anderson, De Palma and Thisse, 1992). When the unit costs are convex and the utility concave with the levels of specific attributes, a perfect Nash equilibrium of attributes and prices choices always exists. One firm chooses the highest possible level of attributes whereas its rival selects a level that equalizes the marginal utility and the marginal cost of the attributes. This outcome is explained notably by the existence of a pricing imitation between firms, already highlighted in the EBA duopoly. When the only fixed costs depend on attributes, additional assumptions on the form of utility and costs functions are required to prove the existence of a global perfect Nash equilibrium. We use the classical linear utility and quadratic fixed costs and show that the differentiation is both horizontal and vertical at the equilibrium.

In order to carry out a normative analysis, the consumer's surplus is defined as the expected utility of consumption of the goods which includes both the specific attributes and the attributes common with the other goods. When costs are exogenous, the equilibrium level of attributes corresponds to the social optimum, as in the logit model. But when the unit costs vary with the attributes, the highest level of attribute at the equilibrium is too high in terms of welfare. In this case, the social optimum corresponds to a purely horizontal differentiation. This sub-optimal excess of differentiation is also true when fixed costs vary with the attributes.

The paper is organized as follows. The properties of the Nash price equilibrium in the EBA duopoly are recalled in section 2. Section 3 compares the corresponding properties of quality choices with those of the vertically differentiated duopolies. In section 4, the choices of attributes of firms are analyzed when costs are exogenous. Section 5 extends this analysis when the unit costs vary with the attributes. Then, section 6 treats the case of attributes-dependent fixed costs. A normative analysis of the attributes choices of firms is carried out in section 7. The last section concludes and proofs of some propositions are carried in the Appendix.

2 Price equilibrium in the EBA duopoly

Suppose that a consumer must choose one product among two that are sold at the same price. Each good i possesses its own *specific attributes* that gives the consumer an utility noted u_i. In this case, a consumer using the EBA heuristic chooses the product i with the following probability, given by Tversky (1972) :

$$P_i = \frac{u_i}{u_i + u_j} \tag{2.1}$$

This probability is thus equal to the utility ratio of the specific attributes of the goods. When u_i is interpreted as the utility of the good i, instead of the utility of the specific attributes of i, this formula recalls that of the Luce model (1959).

However, this equivalence is not true when the prices of the goods, p_i and p_j, are different. Indeed, Rotondo (1986) showed that the price difference of the cheapest good with its rival must be taken into account as a specific attribute in the EBA model. Thus, when $p_i > p_j$, the product j possesses an additional attribute providing an utility $\theta(p_i - p_j)$. The consumers can thus eliminate a good because it does not possess a particular non-price attribute or because its price is too high compared to that of the other good.

In a market with N consumers following EBA, the demand is described by $X_i = NP_i$ and depends on the price hierarchy :

- if $p_i \geq p_j$,

$$X_i = \frac{Nu_i}{u_i + u_j + \theta(p_i - p_j)} \tag{2.2}$$

- if $p_j \geq p_i$,

$$X_i = \frac{N(u_i + \theta(p_j - p_i))}{u_i + u_j + \theta(p_j - p_i)} \tag{2.3}$$

The parameter θ can be seen as the relative importance of the price attribute compared to non-price attributes. This function admits a kink and thus is not strictly concave[2].

Suppose that each firm i bears a unit cost c_i and a fixed cost F_i and chooses the price which maximizes its profit. The existence of a Nash equilibrium in pure strategies has been studied and leads to this proposition :

Proposition 1 (Laurent, 2006a, p 10) *Necessary and sufficient conditions for the existence of a Nash equilibrium in $p_i \geq p_j$, with $i, j \in \{1, 2\}$ and $i \neq j$, are*

$$u_i \geq u_j \tag{2.4}$$

and

$$c_i - c_j \geq \frac{\sqrt{u_i u_j} - u_i}{\theta} \tag{2.5}$$

If this equilibrium exists, then it is unique.

[2]similarly to the "switching costs" model, as in Klemperer, 1984

The equilibrium prices when $p_i > p_j$ are :

$$p_i^* = \frac{u_i + \sqrt{\Delta}}{2\theta} + c_i \tag{2.6}$$

$$p_j^* = \frac{u_i + u_j}{\theta} + c_i \tag{2.7}$$

where $\Delta = u_i^2 + 4u_i(u_i + u_j + \Delta c)$ and $\Delta c = \theta(c_i - c_j)$. Thus, firm i, whose product is the most appreciated by consumers, can choose a highest price than its competitor. This equilibrium exists when $c_i \geq c_j$ but also when c_j is slightly higher than c_i (the costs gap must be weak).

The form of differentiation in the model depends on the utility parameters :
- When $u_i = u_j > 0$, the specific attributes of each good are appreciated in the same way by consumers. When $c_i = c_j = c$, then $p_i = p_j > c$, which is the sign of a differentiation. This configuration, in which all variants have a positive demand when they are sold at the same price, corresponds to a *pure horizontal differentiation*, as in the Hotelling (1919) model.
- When $u_i > 0$ and $u_j = 0$, one of the goods possesses all the specific attributes of the market. Thus, when the prices are identical, all the consumers prefer purchasing the product i rather than j : the existence of such a preference hierarchy is the sign of a *pure vertical differentiation*, as in the models of Mussa and Rosen (1978) or Gabszewicz and Thisse (1979).
- Thus, the general case in which $u_i > u_j > 0$ includes a *double differentiation*: *differentiation is horizontal up to the level* u_j, the goods providing to consumers similar services, then *vertical for a level* $u_i - u_j$, the product i proposing also additional attributes.
- Finally, the Bertrand case is obtained when $u_i = u_j = 0$ et $c_i = c_j = c$: prices become $p_i = p_j = c$.

In the logit model, differentiation is intrinsically horizontal: aspects of vertical differentiation can be integrated but, in this case, prices can no more be computed explicitly. The way of representing differentiation is also very different between these models.

Note now that each price increases with both u_1 and u_2 : each price increases with the *global* degree of differentiation on the market. Moreover, the price of j increases with c_i : indeed, for a particular gap of attribute levels and a particular gap of costs, a fixed and strict gap of prices is required for the existence of the equilibrium. This relation reminds practices of pricing imitation, like those described by Lazer (1957 p. 130-131) : the firm selling the “best quality” good sets a *reference price* on the market (or “focal price”) and the other firm prices at this reference threshold minus a special amount, which depends on the quality gap with the reference firm. Such an explanation also allows to justify the existence of a price equilibrium for some values of c_1 largely higher than c_2.

At the outcome, profits are given by :

$$\Pi_i^* = \frac{Nu_i}{\theta} - F_i \tag{2.8}$$

$$\Pi_j^* = \frac{N(u_i + u_j + \theta(c_i - c_j))(\sqrt{\Delta} - u_i)}{\theta(\sqrt{\Delta} + u_i)} - F_j \tag{2.9}$$

Moreover, $\Pi_i^* > \Pi_j^* \Leftrightarrow \Delta c < u_i - u_j$. Thus, the firm selling the "most appreciated" product (such that $u_i > u_j$) has a highest profit when the gap of differentiation is high enough compared to the gap of costs.

After having reminded the conditions and properties of the price equilibrium, we study if the attributes choices of the firms are consistent with the equilibrium configurations and forms of differentiation exposed.

3 Comparative statics in "qualities"

This section carries on a comparative statics analysis of the EBA profits at the price equilibrium when the utility of the specific attributes varies. The classical properties of quality choices in PVD models are first recalled and then compared to those highlighted in the EBA model.

3.1 Properties of quality choices in vertical differentiation

Models with PVD suppose a non-cooperative two stages game in which two firms choose a quality, produce at zero cost and compete in price. The consumer's willingness to pay the quality (which may depend on consumers' incomes or tastes for quality) is heterogenous in the population and represented by a parameter, say θ, uniformly distributed in a given interval. Each consumer purchases zero or one unit of a good. Firms choose the qualities v_i, that are assumed to verify $v_1 \geq v_2$, and then select their prices, noted p_i.

The various PVD models are generally supposed to be similar in the literature. However, a little difference about the utility obtained by the consumers exists between these structures. This property plays a role in our analysis : for a consumer θ, models a la Mussa-Rosen (1978) (or their derivatives proposed by Tirole, 1988, or Choi and Shin, 1992) suppose that the utility of a product i is $U_i = \theta v_i - p_i$ whereas models a la Gabszewicz-Thisse (1979) (or Shaked and Sutton, 1982) assume that the utility is $U_i = v_i(\theta - p_i)$. In other words, the perception of prices is directly affected by quality in Gabszewicz-Thisse type models but not in Mussa-Rosen ones.

At the price equilibrium, firms realize profits $\Pi_1(v_1, v_2)$ and $\Pi_2(v_1, v_2)$. In this framework, the *existence* of a perfect Nash equilibrium requires that the profits verify the three following properties :

P1 : If $v_1 > v_2$ then $\Pi_1(v_1, v_2) > \Pi_2(v_1, v_2)$.

The firm selling the high quality good realizes a higher profit than its rival.

P2 : $\dfrac{\partial \Pi_1(v_1, v_2)}{\partial v_1} > 0$

The profit of firm 1 increases with the quality of the product sold. While taking as a starting point an interpretation suggested by Beath et al. (1987), this relation can be explained by the combination of two effects : a "*competitive effect*" and a "*competitiveness effect*". The former corresponds to the profit variation induced by a change of the quality gap between products (substitutability between goods). The latter is the profit variation coming from the modification of the position of the firm compared to its rival (performance). In this case, when v_1 increases, the two effects are positive : the competitive effect is positive as the gap $v_1 - v_2$ increases, diminishing the substitutability of the products, and the competitiveness effect is positive because firm 1 improves its position relatively to firm 2.

P3 : $\dfrac{\partial \Pi_2(v_1, v_2)}{\partial v_1} > 0.$

The profit of firm 2 increases with the quality of good 1. Here, the competitive effect is positive for firm 2 as the products are less substituables but the competitiveness effect is negative as the position of firm 2 is deteriorated compared to that of firm 1. This property thus implies that the competitive effect is stronger than the competitiveness effect.

Specific properties of firms' profits when v_2 varies are not required for the proof of existence of the equilibrium but PVD models also verify :

P4 : $\dfrac{\partial \Pi_1(v_1, v_2)}{\partial v_2} < 0$

The profit of firm 1 decreases with the quality of product 2. Indeed, both the competitiveness and the competitive effects are negative as the rivalry between firms increases when the quality gap diminishes.

P5 : $\Pi_2(v_1, v_2)$ is non-monotonic concave or strictly decreasing with v_2

More precisely, the profit of firm 2 is decreasing when the market is covered (every consumer θ decides to purchase). When the market is not covered, the profit is non-monotonic concave in the models a la Mussa-Rosen and decreasing in the models a la Gabszewicz-Thisse. The form of this profit can still be explained by the two effects previously mentioned : the negative competitive effect diminishes the profit of 2, because products are closer substitutes, but the competitiveness effect is positive as the relative position of 2 is improved. Finally, the profit is concave when the competitiveness effect is dominant at first and the competitive effect dominant afterwards. When the competitive effect is always dominant, the profit is strictly decreasing.

3.2 A comparaison with the EBA model

In the EBA framework, each firm can improve the quality of its product by two means : innovation and imitation. For a firm i, the innovation consists in increasing the quantity of specific attributes in its product, which rises u_i. But the "relative" quality of product i may also be improved by adopting some attributes of a competing product. In this case, an increase of quality through the imitation implies a decrease of u_j, as some specific attributes of j becomes common to the various products. In the following sections, we focus our analysis on the choice of innovative attributes. In other words, we make the (sometimes strong) assumption that no imitation is possible : each firm i chooses its own level u_i.

The comparaison between PVD models and the EBA framework may be easily realized in replacing the v_i, corresponding to the "purely vertical" qualities, by the u_i, denoting the "qualities" with double differentiation. The assumption $c_1 = c_2$ is also required. In this case, profits at the price equilibrium in $p_1 \geq p_2$ become :

$$\Pi_1^* = \frac{N}{\theta} u_1$$

$$\Pi_2^* = \frac{N(\sqrt{5u_1^2 + 4u_1u_2} - u_1)(u_1 + u_2)}{\theta(\sqrt{5u_1^2 + 4u_1u_2} + u_1)}$$

The property **P1** is verified in the EBA model, as it is shown in Laurent (2006a, p 13). It is obvious that **P2** is verified and this is also true for **P3** :

$$\frac{\partial \Pi_2^*}{\partial u_1} = \frac{4Nu_1(u_1 + u_2)(\sqrt{\Delta} - u_2)}{\theta\sqrt{\Delta}(\sqrt{\Delta} + u_1)^2} > 0$$

However, the property **P4** does not hold in the EBA model as the profit of firm 1 does not vary with u_2. Such a relation comes from the nature of differentiation by attributes : *an increase of u_2 does not mean that the products are closer substitutes but conversely that the product 2 has additional distinctive attributes.* In other words, an increase of u_2 is the result of an innovation by firm 2. Conversely, an increase of v_2 in a PVD model is assumed to increase the substitutability between goods : the quality improvement is realized through a certain form of imitation of product 1. Concerning firm 1, an increase of v_1 or u_1 is always perceived as an innovation in the two frameworks. Thus, the EBA model seems more consistent : for both goods, an increase of u_i corresponds to a quality improvement by innovation, allowing firms to increase their prices. Consequently, the competitive effect of a raise of v_2 is replaced by a global "*differentiation effect*" of a raise of u_2, already highlighted in the price equilibrium analysis. Whereas the competitive effect can be negative in PVD models, the global differentiation effect is always positive in the EBA model. Finally, an increase of u_2 has a positive differentiation effect and a negative competitiveness effect on Π_1. As each effect cancels the other in this framework, the profit of 1 is constant with u_2.

The property **P5** does also not hold in the EBA framework since Π_2 strictly increases with u_2 :

$$\frac{\partial \Pi_2^*}{\partial u_2} = \frac{4Nu_1(u_1 + u_2)}{\theta\sqrt{\Delta}(\sqrt{\Delta} + u_1)} > 0$$

This outcome derives from the replacement of the competitive effect by the differentiation effect. As these two effects are positive, the profit of 2 increases with the quality of its product.[3]

Finally, the approach followed in the EBA model is complementary with that of the models possessing several dimensions of differentiation, as in Neven and Thisse (1990) or Irmen and Thisse (1998). In these models, the number of characteristics inducing the differentiation is fixed exogenously by the modeler whereas it is determined endogenously by firms' choices of attributes in EBA. Moreover, a change of positioning of a firm in a particular dimension of differentiation is assumed to have no impact on its other positions in these models. This is not true in the EBA model : thus, when u_2 increases, there is a simultaneous increase of the horizontal dimension and a reduction of the vertical dimension.

[3]The difference between the EBA model and the standard PVD models about the properties **P4** and **P5** also opens interesting research prospects. Indeed, the properties from **P1** to **P5** have been frequently used as assumptions in many papers concerning quality choices (for example, Aoki and Prusa, 1997, or Lehman-Grube, 1997), innovation analysis (as Beath and al, 1987 or Rosenkrantz, 1997), and so on. Thus, a simple modification of the properties **P4** and **P5** can allow to study the effects of an additional horizontal dimension of differentiation in all these models.

Despite this difference of structure, some common points exist between the models. In particular, the global differentiation effect highlighted here can be retrieved in the framework of Neven and Thisse. Their model possesses two decision parameters indicating respectively the horizontal and vertical positioning of the firms in these two dimensions. At the outcome, each firm chooses a maximum differentiation in a dimension (the characteristic is called "dominant") and a minimum in the other dimension ("dominated" characteristic). When the vertical characteristic is dominant, prices increase when the varieties are closer, because an effect of central localization dominates the classical effect of price competition. Moreover, when the horizontal characteristic is dominant, the price of the low-quality product increases when the gap of quality diminishes, because the quality improvement weaken the effect of price competition. Thus, in these two cases, the price competitive effect is replaced by an effect increasing the differentiation : the central localization and quality effects of Neven and Thisse can be seen as two facets of the differentiation effect highlighted here.

4 Attributes choice with exogenous costs

In this section, each firm chooses the level of available specific attributes maximizing its profit and the costs are not linked with the attributes. The steps of the considered game are first presented and then the Nash equilibrium is studied.

4.1 The stages of the game

Competition between firms is modelled by a two stages game in pure strategies.

In the *first stage*, each firm i chooses the specific attributes of its product among a set of available binary characteristics noted K_i.[4] We made the assumption that the sets of attributes available to the two firms are disjoint : $K_i \cap K_j = \emptyset$. This assumption means that each firm is sure of the specificity of its attributes. The attributes chosen for i are noted $k_i \subset K_i$.

Suppose now that every set of attributes can be represented by a synthetic indicator, a continuous positive variable $q_i : k_i \to [0; Q_i[$ with $Q_i = q(K_i)$. This index is viewed as a measure of the level of "accessories" of the considered good, that we sometimes call "quality"[5]. Several sets of specific attributes can be associated to a same level q. These indicators allow an easier comparaison of the quality levels between firms than the sets of attributes.

A level of specific attributes q brings the consumers an utility $u : q \to \mathbb{R}^+$. Thus, the buyer of a good i with a set of characteristics k_i gets utility $u_i = u(q_i)$ (with $u(0) = 0$). If firms choose the sets of specific attributes k_1 and k_2 verifying $q_1 = q_2$, the products provide the same utility to the consumers. The function u also verifies $u'(q_i) \geq 0$: any increase of the specific attributes implies an additional utility to the consumer. Finally, we make the classical assumption that the marginal utility of the attributes decreases with the level of attributes : $u''(q_i) \leq 0$ and $\lim\limits_{q_i \to +\infty} u'(q_i) = 0$.

[4]Taking into account continuous characteristics, such as the power of a vehicle, is delicate as they necessarily equip all the products. A firm can not anticipate if the level retained for such an attribute will be higher or lower than that of its rivals. Thus firms can never be sure that such an attribute really provides a comparative advantage to the product.

[5]Remind that this is not identical to the quality in PVD models as the attributes shared by the goods are not integrated

In the *second stage*, the previous quality choices are of a common knowledge and firms compete in prices. The equilibrium prices are given by equations 2.6 and 2.7.

The game is solved by backward induction. For a couple of equilibrium qualities $(q_1^*; q_2^*)$ obtained at the first stage, the price subgame at the second stage is solved by a couple of prices $(p_1^*(q_1^*; q_2^*); p_2^*(q_1^*; q_2^*))$ such that the following inequality holds for each firm i $(i = \{1; 2\})$:

$$\Pi_i(p_i^*, p_j^*; q_1^*, q_2^*) \geq \Pi_i(p_i, p_j^*; q_1^*, q_2^*) \;\; \forall p_i \in [0; +\infty[, \; \forall i, j \in \{1, 2\}, \; i \neq j$$

At the first stage, the equilibrium of the quality subgame is then solved by a quality couple $(q_1^*; q_2^*)$ such that the quality chosen by i satisfies :

$$\widehat{\Pi_i}(q_i^*, q_j^*) \geq \widehat{\Pi_i}(q_i, q_j^*) \;\; \forall q_i \in [0; Q_i[, \; \forall i, j \in \{1, 2\}, \; i \neq j$$

with $\widehat{\Pi_i}(q_i^*, q_j^*) = \Pi_i(p_1^*(q_i^*, q_j^*), p_2^*(q_i^*, q_j^*); q_i^*, q_j^*)$ corresponding to the profit evaluated at the second stage.

A subgame perfect Nash equilibrium is defined by a couple of equilibrium qualities $(q_1^*; q_2^*)$ and a couple of equilibrium prices $(p_1^*(q_1, q_2), p_2^*(q_1, q_2))$ for all qualities (q_1, q_2). Such an equilibrium is now used to analyze the quality choices.

4.2 Equilibrium with exogenous costs

In this section, unit and fixed costs are not linked with the qualities and are assumed to verify $c_1 = c_2$ and $F_1 = F_2$. At the end of the first stage, suppose that the quality choices verify the conditions (2.4) and (2.5). In this case, if an equilibrium exists at the second stage, then the equilibrium prices are given by (2.6) and (2.7).

But as previously mentioned, each firm's profit increases with the level of attributes : thus, each firm chooses the highest quality possible. However, the existence of a perfect Nash equilibrium is guaranteed only if the equilibrium levels of attributes effectively verify the condition (2.4), which imposes the inequality $Q_1 \geq Q_2$. These conclusions are resumed here :

Proposition 2 *When the costs are independent of the attributes, there exists two subgame perfect Nash equilibria in $p_i \geq p_j$ with $i, j \in \{1, 2\}$, $i \neq j$, $Q_i \geq Q_j$ and which differ only by the identity of the firms. The highest possible qualities are selected.*

This result is similar to that obtained by Anderson, De Palma and Thisse (1992, p 237) with the logit model : the EBA model also respects the *principle of maximum differentiation* because each firm has interest in pushing as far as possible its avantage in terms of differentiation. This differentiation is horizontal for a level Q_j and vertical for a level $Q_i - Q_j$.

The results obtained in this section are strongly linked to the assumption of exogenous costs, that we question in the following sections. As in Anderson, De Palma et Thisse, we distinguish between unit costs and fixed costs varying with the qualities.

5 Choice of attributes with endogenous unit costs

After introducing the assumptions on unit costs, this section studies the quality equilibrium with endogenous unit costs.

5.1 Assumptions on the unit costs

This section keeps the function $u_i = u(q_i)$ and its properties but it is now assumed that a rise of the attributes level increases the unit costs of production. Fixed costs are set to zero. Thus, each firm possesses a set of available specific new attributes, discovered in the previous periods by its applied research service. Its objective is to choose the attributes which will be finally added to the product. This kind of decision is generally taken at the end of the innovation phase after a discussion between the development and marketing services.

Thus, the unit cost of each firm is modelled by a same function $c : q \to \mathbb{R}^+$ with $c(0) = 0$. This unit cost of a firm i is supposed constant with the quantity produced but increases in a convex way with quality : $c'(q_i) > 0$ and $c''(q_i) \geq 0$. The attributes realizing the best balance between the cost and the utility provided are chosen in first and the less "efficient" attributes are selected afterwards.

Finally, we suppose that the maximum available level of specific attributes is identical for the two firms: $Q_1 = Q_2 \to +\infty$. That means that we can set $\lim\limits_{q_i \to Q_i} u(q_i) = u_\alpha$ where u_α is a "reservation utility" threshold reached by the consumers when the level of attributes is maximum.

5.2 Perfect equilibrium

We study the existence of a perfect equilibrium in which firms choose quality and then prices when the unit costs are endogenous. At the first stage, the quality choices are resumed in this proposition :

Proposition 3 *When the unit costs depend on the attributes, there exists two perfect Nash equilibria verifying $p_i \geq p_j$ and which differ only by the identity of the firms. The firm i chooses the highest quality possible such that $u'(q_i) = 0$. The firm j chooses a level of attributes such that the marginal weighted utility of the consumer is equal to the marginal cost of production : $u'(q_j)/\theta = c'(q_j)$.*

Proof: this proof is presented in Appendix 10.1.

For firm i, whose product is the most appreciated, the introduction of a unit cost depending on the quality does not modify the result obtained in the previous section, as the highest level of attributes is retained. This choice of i is a result of the weak nature of the price Nash equilibrium in the model. Indeed, i being a "reference" firm, any increase of its unit costs also induces a rise of the price of its rival and the position of i in the market is not weakened. As its profit is invariant with c_i, this "reference firm" is incited to choose the highest level of quality, whatever the cost beard. However, as the form of Π_i depends of this "reference price" effect and as this outcome is not affected by the level of θ, it is clear that the outcome is not a disturbing consequence of the absence of outside option. The choice of the highest level of attributes for i is clearly not a realistic outcome. But this outcome should not create doubt on the relevance of the model because it depends crucially of the simplifying assumption of no-imitation we use in this paper. Indeed, when a threat of imitation exists on the market, firm i is incited to diminish its level of specific attributes and a more reasonable outcome is obtained (see Laurent, 2006b).

The choice of attributes by firm j is affected when the unit costs become endogenous : a lower quality is chosen. The equilibrium level of attributes decreases with θ. In particular, if $\theta \to +\infty$ (or if the costs are very convex), the firm chooses not to develop specific attributes at all, which leads to a purely vertical differentiation market structure. However, in the general case, the two sides of differentiation exist at the equilibrium : the differentiation is horizontal for a level q_j and vertical for a level $q_i - q_j$.

The existence of asymmetric qualities is a standard outcome in PVD models (see Motta, 1993) but not in the standard logit. However, there exists some common properties between the logit and the EBA models when the unit costs increase with the quality : a variation of the number of consumers N or an increase of the constant unit or fixed costs of production have no impact on the quality choices.

6 Choice of attributes with endogenous fixed cost

First, this section presents the assumptions on fixed costs. The existence of a subgame perfect equilibrium with choice of attributes and price competition is then established.

6.1 Assumptions on fixed costs

Whereas the assumption of endogenous unit cost follows the perspective of a product development service, the assumption of an endogenous fixed costs rather corresponds to the upstream point of view of an applied research service. The quantity of new attributes provided by this service depends on the investment realized in research, which induces a fixed cost.

This section keeps the assumption $Q_1 = Q_2 = +\infty$ and the properties of the function $u_i = u(q_i)$ presented previously. But we now suppose that each unit cost is constant whereas each fixed costs is a function $F : q \to \mathbb{R}^+$, beard identically by each firm, with $F(0) = 0$. This function is increasing and convex with the level of attributes chosen for the product i : $F'(q_i) > 0$ and $F''(q_i) \geq 0$. Such an assumption is relatively frequent in the literature : the increase of the number of specific attributes makes them more and more costly to discover.

However, the proof of equilibrium existence can frequently not be realized with functions of a general form. That is why an additional assumption may be required :

Assumption 1 : The utility has a linear form $u_i = q_i$ and the fixed costs are quadratic $F_i = q_i^2$.

The functions assumed here are both meaningful and quite standard in the literature : for instance, a linear utility is used by Anderson, De Palma and Thisse (1992) and a quadratic fixed cost function is assumed by Ronnen (1991), Motta (1993) or Pepall and Richards (1994).

As previously, the existence of a perfect equilibrium in which firms choose the levels of attributes and then the prices is studied. In order to guarantee the existence of a price equilibrium at the second stage, whatever the parameters of the utility function, the unit costs verify $c_1 = c_2 = c$.

6.2 Existence and uniqueness of the equilibrium

If the Nash equilibrium at the second stage verifies $p_i \geq p_j$, the equilibrium prices are given by the equations (2.6) and (2.7) and the profits are :

$$\Pi_i(q_i) = \frac{N}{\theta}u(q_i) - F(q_i) \text{ and } \Pi_j(q_j) = \frac{N(u(q_i)+u(q_j))(\sqrt{\Delta}-u(q_i))}{\theta(u(q_i)+\sqrt{\Delta})} - F(q_j)$$

with $\Delta = u(q_i)^2 + 4u(q_i)(u(q_i)+u(q_j))$

The study of the optimal quality choices of the firms leads to the following proposition :

Proposition 4 *When $q_i \geq q_j$ for $i, j \in \{1,2\}$ and $i \neq j$, the function Π_i is concave and the maximum verifies $\frac{u'(q_i)}{\theta} = \frac{F'(q_i)}{N}$. The first derivatives of Π_j is zero if $\frac{u'(q_j)}{\theta}\frac{4u_i x}{\sqrt{\Delta}(\sqrt{\Delta}+u_i)} = \frac{F'(q_j)}{N}$. The Assumption 1 guarantees both the concavity of Π_j and the existence of a global perfect Nash equilibrium of attributes-prices choices.*

Proof : presented in Appendix 10.2.

Here again, the identity of the firm choosing the highest level of attributes is unknown at this local equilibrium. The differentiation is both horizontal and vertical. The level of attributes chosen by firm i equalizes the marginal utility weighted by θ and the additional fixed cost beard for the product. The level chosen by firm j is lower. The two levels decrease with θ and increase with N, the number of consumers, as in the logit model. Indeed, the marginal investment in quality is more profitable the more consumers they are (Anderson, De Palma et Thisse 1992, p 245). When the Assumption 1 is verified and when $\theta = N = 1$, the levels of attributes at the equilibrium are $q_i = 0.5$ and $q_j = 0.317954$. Note that with these functions, firm j realizes a higher profit than its rival.

After this descriptive study of the quality choices in the EBA model, the following section analyzes the decision of the firms under a normative angle.

7 Equilibrium and socially optimal differentiation

The previous sections proved the existence of a perfect Nash equilibrium in which firms first choose their level of attributes and then compete in price. In this section, we determine if these choices are beneficial for the consumers and optimal for the social welfare.

7.1 Product differentiation and consumer surplus

The impact of the degree of differentiation on the consumer surplus is generally ambiguous. On the one hand, a differentiated product allows for a better match with consumers' tastes. On the other hand, the differentiation increases firms' market powers, which raises the price paid by the consumers. The preferences of the consumers are obviously supposed inobservable by firms, preventing any perfect price discrimination.

At this stage, note that the welfare question has already been treated in an other discrete choice model, the multinomial logit (Anderson, De Palma et Thisse, 1992, p 202). In this model, the consumer surplus equals the multiplication of the number of consumers (N) and the utility of the option chosen by each consumer. This utility includes a deterministic part depending on the quality and on the price of the good and a random component described by :

$$U_\varepsilon = n \int_{-\infty}^{+\infty} x f(x) [F(x)]^{n-1} dx \tag{7.1}$$

In this equation, n denotes the number of firms, x the contribution of the random term to the surplus, $f(x)$ the probability of selecting this random term and $[F(x)]^{n-1}$ the probability that this random term is the highest for the considered consumer. The choice probabilities thus play an essential role on the computation of the surplus.

This exact formula can not be used in the EBA model as it is the decision rule that is random instead of the utility. But we follow a similar method. The surplus of the consumer is defined as the product of the choice probability of an option and of the utility provided by this option. The surplus is thus equal to the expected utility of the consumer. Nevertheless, the formula requires to be able to define a "global" utility for each option. Such an utility does not intervene in the choice process of the good because the only utility of the specific attributes is taken into account. Thus, there exists a kind of disconnection between the choices of the consumers and their evaluation of these choices, between the descriptive part of the model and its normative part. When people choose among options, the cognitive cost of the decision plays a role. But in the evaluation of this decision, the consumer only considers the consistence of the decision with its preferences. Indeed, it is not so rare that people regret their decision after the purchase, even if the choice has been realized in a context (tiring, numerous and complex options) in which it is preferable to use a heuristic as the EBA one.

We suppose that the global utility of a good depends on the set of its attributes, which are specific or common with some other goods, and of its price. As previously defined, the specific attributes k_i of the goods, for $i = \{1, 2\}$, provide the consumers the utility $u_i = u(q_i)$ where q_i denotes the level of these attributes. The set of the attributes k_0 common to all the goods of the choice set A are now assumed to give the consumers the utility $u_0 = u(q_0)$. These utilities are weighted by the coefficient θ measuring the relative importance of these non-price attributes relatively to the price of the good (the highest θ is, the less these attributes are important). Consequently, the net utility of a good i is :

$$U_i = \frac{u_0 + u_i}{\theta} - p_i \tag{7.2}$$

The surplus of the N consumers can now be formulated as $SC = \sum_{i \in A} N P_i U_i$. Suppose (here and for the following sections) that the necessary and sufficient conditions for the existence of a Nash price equilibrium in $p_1 \geq p_2$ are verified when the product 1 possesses the highest level of attributes. In this case, the consumer surplus at the outcome is :

$$\begin{aligned} SC &= \frac{2Nu_1}{u_1 + \sqrt{\Delta}} \left(\frac{u_0}{\theta} + \frac{u_1 - \sqrt{\Delta}}{2\theta} - c_1 \right) + \frac{N(\sqrt{\Delta} - u_1)}{u_1 + \sqrt{\Delta}} \left(\frac{u_0 - u_1}{\theta} - c_1 \right) \\ &= \frac{N}{\theta} \left(u_0 - \theta c_1 + \frac{2u_1(u_1 - \sqrt{\Delta})}{u_1 + \sqrt{\Delta}} \right) \end{aligned} \tag{7.3}$$

with still $\Delta = u_1^2 + 4u_1 x$ and $x = u_1 + u_2 + \theta(c_1 - c_2)$.

We see now if the differentiation is beneficial for the consumers or if the negative effect of market power rise is dominant. First note that the consumer surplus strictly increases with the utility of the common attributes u_0 and decreases with the importance of the price attributes θ. Thus, the higher the disutility from the loss of a monetary amount is, the lower the surplus obtained by the consumption of a unit of good is.

We study how the surplus varies with u_1 and u_2 ?

$$\frac{\partial SC}{\partial u_1} = \frac{-4N(x-u_1)}{\theta\sqrt{\Delta}(u_1+\sqrt{\Delta})^2} \tag{7.4}$$

$$\frac{\partial SC}{\partial u_2} = \frac{-8Nu_1^3}{\theta\sqrt{\Delta}(u_1+\sqrt{\Delta})^2} < 0 \tag{7.5}$$

Thus, the surplus decreases with u_2 but the derivative with u_1 is negative if and only if :

$$x - u_1 \geq 0 \Leftrightarrow \Delta c \geq -u_2 \tag{7.6}$$

Suppose that this condition is verified, as it is generally true. In this case, the surplus of all buyers decrease with the levels of attributes chosen by each seller : a minimum differentiation between firms is the better situation for consumers. This outcome may seem surprising compared to the properties of the consumer surplus in PVD models. In these models, the surplus of the high-quality good buyers increases with the quality whereas the converse relation is true for low-quality good buyers. But, in fact, the two outcomes are equivalent : the consumer surplus decreases *with the differentiation on the market.* Indeed, in pure vertical differentiation, an increase of differentiation corresponds to a rise of the gap between the qualities whereas in this model with double differentiation, an increase of any attributes level strengthens the differentiation. In this sense, note that the probability of verifying condition 7.6 decreases with the horizontal dimension of differentiation u_2.

7.2 Levels of attributes and social welfare

This section defines the welfare function and analyzes if the quality choices of the firms are optimal in terms of global surplus. This section considers the case of exogenous costs and introduces endogenous unit and fixed costs afterwards.

7.2.1 Attributes choices and welfare with exogenous costs

As usual, the welfare function is the sum of producers and consumers surplus : $W = \Pi_1 + \Pi_2 + SC$. At the price equilibrium, this function equals :

$$\begin{aligned} W &= \frac{N}{\theta}\left(u_0 - \theta c_1 + \frac{2u_1(u_1-\sqrt{\Delta})}{u_1+\sqrt{\Delta}}\right) + \frac{N}{\theta}u_1 + \frac{N(u_1+u_2+\Delta c)(\sqrt{\Delta}-u_1)}{\theta(\sqrt{\Delta}+u_1)} \\ &= \frac{N}{\theta}\left(u_0 - \theta c_1 + u_1 + \frac{(\sqrt{\Delta}-u_1)(x-2u_1)}{\sqrt{\Delta}+u_1}\right) \end{aligned} \tag{7.7}$$

Suppose that $Q_1 \geq Q_2$, $c_1 = c_2$ and $F_1 = F_2$ which guarantees that the perfect Nash equilibrium exists (and that condition 7.6 holds). In this framework, the differentiation of the products has two opposite effects on the welfare : profits increase with each u_i whereas consumers surplus decrease with them.

For *product* 1, the slope of the welfare function is :

$$\frac{\partial W}{\partial u_1} = \frac{2Nu_1[u_1(\sqrt{\Delta} + 5u_1 + 4u_2) + 2u_2(u_1 - u_2)]}{\theta\sqrt{\Delta}(\sqrt{\Delta} + u_1)^2} > 0 \tag{7.8}$$

The highest possible value of u_1 is also the socially optimal level. As shown in the section 4.2, this value is precisely that chosen by firm 1 : the quality equilibrium for this firm corresponds to the social optimum.

For *product* 2, the variation of the welfare with u_2 is :

$$\frac{\partial W}{\partial u_2} = \frac{4Nu_1(u_1(u_2 - u_1) + (u_1 + u_2)\sqrt{\Delta})}{\theta\sqrt{\Delta}(u_1 + \sqrt{\Delta})^2} > 0 \tag{7.9}$$

Thus, the welfare function increases with u_2 and the socially optimal level of attributes is, here again, the highest possible. This optimum is also chosen by firm 2 at the equilibrium.

These results are synthesized here :

Proposition 5 *When the costs are symmetric and exogenous with the attributes, the equilibrium levels of attributes, traducing a maximum differentiation, are also socially optimal.*

This result is equivalent to that obtained by Anderson, De Palma and Thisse (1992) with the logit model. Thus, when the utilities increase, the profit gain of firms overcome the surplus loss of consumers. We study now if the equilibrium is still socially optimal when costs depend on qualities.

7.2.2 Social optimum with quality-dependent unit costs

The utility and costs functions considered in this section are supposed to be identical to those of the section 5 which guarantees the existence of the perfect Nash equilibrium.

We initially study which level of attributes q_2 maximizes the social welfare :

$$\frac{\partial W}{\partial q_2} = \frac{4Nu_1(u'(q_2) - \theta c'(q_2))(u_1(x - 2u_1) + x\sqrt{\Delta})}{\theta\sqrt{\Delta}(u_1 + \sqrt{\Delta})^2} \tag{7.10}$$

Thus, the derivative is zero when $u'(q_2) = \theta c'(q_2)$ and the concavity of the welfare function is verified :

$$\left.\frac{\partial^2 W}{\partial q_2^2}\right|_{\frac{\partial W}{\partial q_2}=0} = \frac{4Nu_1(u''(q_2) - \theta c''(q_2))(u_1(x - 2u_1) + x\sqrt{\Delta})}{\theta\sqrt{\Delta}(u_1 + \sqrt{\Delta})^2} \leq 0 \tag{7.11}$$

The socially optimal level of attributes thus verifies the equality between the weighted marginal utility of these attributes and the marginal cost beard by firm 2. As previously seen, this level is also the same as the equilibrium level of the firm 2.

The level of specific attributes q_1 which maximizes the welfare is :

$$\begin{aligned}\frac{\partial W}{\partial q_1} = \frac{N}{\theta}\Bigg(&u'(q_1) - \theta c'(q_1) - \frac{4u_1x(u'(q_1) - \theta c'(q_1))}{(u_1 + \sqrt{\Delta})^2} \\ &+ \frac{4u_1(x - 2u_1)(u_1(u'(q_1) + \theta c'(q_1)) - u'(q_1)x)}{\sqrt{\Delta}(u_1 + \sqrt{\Delta})^2}\Bigg)\end{aligned} \tag{7.12}$$

As the optimal level of attributes for product 2 is $u'(q_2) = \theta c'(q_2)$, this derivative is zero for $u'(q_1) = \theta c'(q_1)$ which implies that $q_1 = q_2$. Indeed, the derivative for this value is given by

$$\frac{\partial W}{\partial q_1} = \frac{N}{\theta}\left(\frac{-4u_1 u'(q_1)(x-2u_1)^2}{\sqrt{\Delta}(u_1+\sqrt{\Delta})^2}\right)$$

and as the costs and utility functions are identical between the firms, we have $x-2u_1 = u_2-u_1+c_1-c_2 = 0$. This level is a maximum as the welfare function is concave :

$$\left.\frac{\partial^2 W}{\partial q_1^2}\right|_{\frac{\partial W}{\partial q_1}=0} = \frac{N}{\theta}\frac{2u_1(u''(q_1)-c''(q_1))}{u_1+\sqrt{\Delta}} \leq 0 \tag{7.13}$$

Thus, at the equilibrium, firm 1 chooses a level of attributes that is too high compared to the socially optimal threshold. These conclusions are resumed here :

Proposition 6 *When the unit costs depend on attributes, the socially optimal level of attributes of firms i and j verify $q_i = q_j = q$ and $u'(q) = \theta c'(q)$. At a perfect equilibrium with $p_i \geq p_j$, the level of attributes chosen by j also maximizes the welfare whereas the level selected by i is too high compared to the social optimum.*

The socially optimal qualities are similar to those obtained in the logit framework by Anderson, De Palma and Thisse (1992, p 242) despite the asymmetric structure of the EBA model. However, contrary to the logit outcome, the equilibrium qualities are not optimal in the EBA model : the quality chosen by firm 1 is too high.

The vertical dimension of product differentiation is the main cause behind increasing prices : indeed, the "high quality" firm does not carry attention to the additional cost implied by an increase of its quality, as this rise will also increase the price of the other firm due to the "reference price" effect. This global increase of prices diminishes drastically the consumers surplus and such overly high quality level generates a global surplus loss. Thus, the social optimum is reached when the differentiation is purely horizontal.

We study now if this outcome is preserved in the case of endogenous fixed costs.

7.2.3 Social optimum with quality-dependent fixed costs

To guarantee the existence of a quality equilibrium, we take again the Assumption 1 of the section 6.1 (linear utility and quadratic fixed cost). Unit costs are still assumed identical.

The first and second derivatives of the welfare function with q_1 and q_2 are :

$$\frac{\partial W}{\partial q_1} = \frac{N}{\theta}\left(\frac{2q_1^2(\sqrt{\Delta}+9q_2)}{\sqrt{\Delta}(\sqrt{\Delta}+q_1)^2}\right) - 2q_1 \tag{7.14}$$

$$\frac{\partial^2 W}{\partial q_1^2} = \frac{N}{\theta}\left(\frac{2q_1^2 q_2[4(\sqrt{\Delta}+9q_2)\sqrt{\Delta} - 9(5q_1+2q_2)(\sqrt{\Delta}+q_1)]}{\sqrt{\Delta}(\sqrt{\Delta}+q_1)^3}\right) - 2 \tag{7.15}$$

$$\frac{\partial W}{\partial q_2} = \frac{N}{\theta}\left(\frac{4q_1(\sqrt{\Delta}(q_1+q_2) - q_1(q_1-q_2)}{\sqrt{\Delta}(\sqrt{\Delta}+q_1)^2}\right) - 2q_2 \tag{7.16}$$

$$\frac{\partial^2 W}{\partial q_2^2} = \frac{N}{\theta}\left(\frac{8q_1^3[(\sqrt{\Delta}+q_1)(5q_1+4q_2) + (q_1-q_2)(3\sqrt{\Delta}+u_1)]}{\sqrt{\Delta}(\sqrt{\Delta}+q_1)^3}\right) - 2 \tag{7.17}$$

with $\Delta = q_1^2 + 4q_1(q_1+q_2)$.

Under the assumption 1, there exists a levels of attributes (q_1^W, q_2^W) such that the first order conditions are verified. For these levels, the welfare function is concave with q_2 in the interval $[0, q_1]$ and with q_1 in the interval $[q_2, +\infty]$. The optimum reached is defined by the following proposition.

Proposition 7 *When the utility is linear and fixed costs quadratic with the attributes, the socially optimal level of attributes of firms i and j verify $q_i = q_j = 1/4$. At the perfect equilibrium in $p_i \geq p_j$, the levels of attributes chosen by i and j are too high compared to social optimum.*

Here again, the social optimum is reached when differentiation is purely horizontal. Introducing a vertical product differentiation in a horizontally differentiated market increases the power of the firms and diminishes the welfare. But, contrary to the case of endogenous unit costs, the level of attributes chosen by firm 2 is also too high compared to the socially optimal level. There is too much vertical and horizontal differentiation in this framework.

8 Conclusion

The properties of the profits have been highlighted in the EBA model, in which differentiation could be both horizontal and vertical. In comparative statics, the profit of each firm increases when the level of innovative attributes of its product increases : this property is similar to those obtained in PVD models. But when the lowest level of attributes increases, the differentiation is strengthened in the EBA model whereas an increase of the lowest quality weakens the differentiation in a PVD duopoly. This conclusion opens prospects in the analysis of innovation, as a horizontal dimension of differentiation can be easily introduced in models that usually consider only a vertical one.

When unit costs are exogenous with the levels of innovative attributes, firms choose the highest possible quality, which is also the social optimum. This result is similar to that obtained in the logit model. The existence of a perfect Nash equilibrium with qualities and then prices choices was also proven when unit costs depend on the level of innovative attributes. In this framework, differentiation is generally both vertical and horizontal. A pure vertical differentiation may happen as a special case when costs of innovation are strongly convex or when price attributes are very important for consumers compared to non-price attributes. At the equilibrium, “low quality” firm selects a level which equate the marginal utility and the marginal cost of innovative attributes. Its rival chooses the highest possible level of attributes. The proof of existence of the perfect equilibrium when fixed costs are endogenous can not be realized without particular functions : we retained the classical assumptions of linear utility and quadratic fixed costs. In this case, differentiation is also horizontal and vertical.

When unit or fixed costs are attributes-dependent, the social optimum corresponds to a pure horizontal differentiation. In the EBA model, the introduction of a vertical differentiation in addition to an horizontal one strengthen the market power of the firms. Consequently, prices raise and consumers surplus diminishes. This negative effects dominates the profit gain of firms. The equilibrium levels of attributes are too high compared to the social optimum : there is too much horizontal differentiation with attributes-dependent unit costs and too much horizontal and vertical differentiations with attributes-dependent fixed costs.

But this two-stage game with innovation and then price competition is not completely convincing in the analysis of the firms' choices of attributes. In the one hand, when unit costs are endogenous, one firm chooses the highest level of attributes, which is an unrealistic outcome. In the other hand, the purely horizontal differentiation can not be a result of firms' choices whereas this kind of differentiation seems to exist in some markets. These problems are clearly linked with the absence of product imitation in the model. Thus, introducing an imitation in a three-stage game is a natural extension of this work (Laurent, 2006b).

9 References

Anderson S.J., De Palma A. and Thisse J-F. (1992), *Discrete choice theory of product differentiation*, Cambridge, MIT Press.

Aoki R. and Prusa T.J. (1997), "Sequential versus simultaneous choice with endogenous quality", *International Journal of Industrial Organization*, 15, 103-121.

Beath J., Katsoulacos Y. and Ulph D. (1987), "Sequential Product Innovation and Industry Evolution", *Economic Journal*, 97, 32-43.

Choi C.J. and Shin H.S. (1992), "A Comment on a Model of Vertical Product Differentiation", *Journal of Industrial Economics*, 40, 229-231.

Gabszewicz J.J. and Thisse J-F. (1979), "Price Competition, Quality and Income Disparities", *Journal of Economic Theory*, 20, 340-59.

Hotelling H. (1929), "Stability in Competition", *Economic Journal*, 39, 41-57.

Irmen A. and Thisse J-F. (1998), "Competition in multi-characteristics. spaces : Hotelling was almost right", *Journal of Economic Theory*, 78, 76-102.

Klemperer P. (1984), "Markets with Consumer Switching costs", *The Quarterly Journal of Economics*, 102, 375-394.

Laurent R.-A. (2006a), "Differentiated duopoly with 'elimination by aspects'", working paper PSE 2006-17.

Laurent R.-A. (2006b), "Imitation and innovation in the 'elimination by aspects' duopoly", mimeo.

Lazer W. (1957), "Price Determination in the Western Canadian Garment Industry", *The Journal of Industrial Economics*, 5, 124-136.

Lehmann-Grube U. (1997), "Strategic Choice of Quality When Quality is Costly: The Persistence of the High-Quality Advantage", *RAND Journal of Economics*, 28-2, p 372-384.

Motta M. (1993), "Endogenous quality choice : Price vs. Quantity Competition", *The Journal of Industrial Economics*, 41, 113-131.

Mussa M. and Rosen S. (1978), "Monopoly and Product Quality", *Journal of Economic Theory*, 18, 301-17.

Neven D., Thisse J.F. (1990), "On Quality and Variety in Competition", in Gabszewicz J.J.,Richard J.F., Wosley L.A. eds., *Decision Making: Games, Econometrics and Optimization*, North Holland, Amsterdam, 175-199.

Pepall L. M. and Richards D. J. (1994), "Innovation, Imitation, and Social Welfare", *Southern Economic Journal*, 60, 673Ũ684.

Ronnen U. (1991), "Minimum Quality Standards, Fixed Costs, and Competition", *The RAND Journal of economics*, 22, 490-504.

Rosenkrantz S. (1997), "Quality Improvements and the Incentive to Leapfrog", *International Journal of Industrial Organization*, 15, 243-61.

Rotondo J. (1986), "Price as an Aspect of Choice in EBA", *Marketing Science*, 5, special issue on Consumer Choice Models, 391-402.

Shaked A. and Sutton J. (1982), "Relaxing Price Competition through Product Differentiation", *Review of Economic Studies*, 69, 3-13.

Tirole J. (1988), *The Theory of Industrial Organization*, Cambridge, MIT Press.

Tversky A. (1972a), "Elimination by aspects : A Theory of Choice.", *Psychological Review*, 79, p. 281-299.

Tversky A. (1972b), "Choice by Elimination", *Journal of mathematical psychology*, 9, p. 341-367.

10 Appendix : proofs on the attributes choice

10.1 Equilibrium with endogenous unit costs

The proof of proposition 3 is presented here. Suppose that the firm with the most appreciated product is firm 1 : $q_1 \geq q_2$. The local maximum in the interval of definition is highlighted and we show that this maximum is also global for all the possible vectors of qualities.

10.1.1 Determination of the local maximum

When $q_1 \geq q_2$, the equilibrium prices at the end of the second stage are $p_1^* = c(q_1) + \frac{u(q_1)}{2\theta} + \frac{\sqrt{\Delta}}{2\theta}$ and $p_2^* = \frac{u(q_1)+u(q_2)}{\theta} + c(q_1)$ with $\Delta = u(q_1)^2 + 4u(q_1)x$ and $x = u(q_1) + u(q_2) + \theta(c(q_1) - c(q_2))$

For *firm 1*, the first order condition is given by :

$$\frac{\partial \Pi_1}{\partial q_1} = \frac{N}{\theta} u'(q_1) = 0 \tag{10.1}$$

At the equilibrium, the second order condition is :

$$\left.\frac{\partial^2 \Pi_1}{\partial q_1^2}\right|_{\frac{\partial \Pi_1}{\partial q_1}=0} = \frac{N}{\theta} u''(q_1) \leq 0 \tag{10.2}$$

The assumptions on the utility function guarantee that this condition always holds. Firm 1 thus chooses q_1^c such that $u'(q_1^c) = 0$: the highest available level of attributes is retained.

For *firm 2*, the following derivative is obtained after simplifications:

$$\frac{\partial \Pi_2}{\partial q_2} = \frac{4Nu_1 x(u'(q_2) - \theta c'(q_2))}{\theta\sqrt{\Delta}(\sqrt{\Delta} + u_1)} \tag{10.3}$$

It equals zero for a value q_2^* verifying $u'(q_2^*) = \theta c'(q_2^*)$. At the equilibrium, the second order condition is :

$$\left.\frac{\partial^2 \Pi_2}{\partial q_2^2}\right|_{\frac{\partial \Pi_2}{\partial q_2}=0} = \frac{4Nu_1 x(u''(q_2) - \theta c''(q_2))}{\theta\sqrt{\Delta}(\sqrt{\Delta} + u_1)} \leq 0 \tag{10.4}$$

The profit is thus concave if $u''(q_2) \leq \theta c''(q_2)$. This condition always holds under the assumptions of concave utilities and convex unit costs.

Finally, the levels of attributes verify $q_1^c > q_2^c$ which implies $u(q_1) \geq u(q_2)$ and $c(q_1) \geq c(q_2)$. Consequently, the necessary and sufficient conditions (2.4) and (2.5) to the existence of an equilibrium in $p_1 \geq p_2$ at the second stage hold. If this local maximum is also global, it constitutes a perfect subgame Nash equilibrium.

10.1.2 Existence of a global maximum

It is proven here that the firms have no interest in deviating from the local maximum, which is also a global maximum.

We initially consider the case of *firm 1* and prove that no level of attributes q_1^{cc} belonging to the interval $q_1^{cc} \leq q_2^c$ verifies $\Pi_1(q_1^c) < \Pi_1(q_1^{cc})$. As $q_1^c = +\infty$, the reference profit is $\Pi_1^c(q_1^c) = \dfrac{N}{\theta} u_\alpha$. If q_1^{cc} exists, then the profit of 1 is similar to Π_2^c : $\Pi_1^{cc}(q_1^{cc}) = \dfrac{Nx(\sqrt{\Delta} - u_2^c)}{\theta(u_2^c + \sqrt{\Delta})}$ with $\Delta = (u_2^c)^2 + 4u_2^c x$ and $x = u(q_1^{cc}) + u_2^c + \theta(c_2^c - c(q_1^{cc}))$. The first derivative looks like equation (10.3) and, consequently, the optimal level of attributes is given by $u'(q_1^{cc}) = \theta c'(q_1^{cc})$ (symmetrically, the second order condition still holds, as in equation (10.4)).

As the utility and cost functions are identical between firms, $q_1^{cc} = q_2^c$ and thus $c_1^{cc} = c_2^c$ and $u_1^{cc} = u_2^c$. We fix this level of utility to u_β. When firm 1 deviates from the local maximum, it chooses the same level of attributes than firm 2 and for this value $\Pi_1^{cc}(q_1^{cc}) = \dfrac{N}{\theta} u_\beta$. But as $q_2^c \leq q_1^c$, we have $u_\beta \leq u_\alpha$ and thus $\Pi_1^c \geq \Pi_1^{cc}$. Firm 1 can never improve its profit by deviating from the local maximum.

Consider now the case of *firm 2*. We study if it exists a level q_2^{cc} belonging to the interval $q_2^{cc} \geq q_1^c$ and verifying $\Pi_2(q_2^c) < \Pi_2(q_2^{cc})$. The reference profit of 2 is given by $\Pi_2^c(q_2^c) = \dfrac{Nx(\sqrt{\Delta} - u(q_1^c))}{\theta(u(q_1^c) + \sqrt{\Delta})}$. If firm 2 deviates from the local maximum, the level of attributes chosen can nevertheless not be strictly higher than that of firm 1 since $q_1^c = Q_1 = Q_2$: firm 2 would necessarily select $q_2^{cc} = q_1^c$. In this case, firm 2 realizes a profit $\Pi_2^{cc}(q_2^{cc}) = \dfrac{N}{\theta} u(q_1^c)$.

The comparaison of profits leads to the condition :

$$\Pi_2^{cc} \leq \Pi_2^c \Leftrightarrow \theta c(q_1^c) - u(q_1^c) \geq \theta c(q_2^c) - u(q_2^c) \tag{10.5}$$

But this inequality is always true because of the concavity of utility and the convexity of unit cost. Indeed, with these type of functions :

$$\frac{d(\theta c(q) - u(q))}{dq} > 0 \ \forall q \geq \overline{q} \ \text{ with } \ \overline{q} > 0 \ \text{ such that } \ \theta c'(\overline{q}) = u'(\overline{q}) \tag{10.6}$$

As we previously showed that q_2^* verified this last equality, we have $q_2 = \overline{q}$. Thus, inequality (10.5) is always verified because $q_1^c > q_2^c$ and firm 2 has no interest in deviating from the local maximum.

As no firm has an interest in deviating and the levels of attributes at the first stage verify the necessary and sufficient conditions for the existence of a price Nash equilibrium at the second stage, we have a perfect Nash equilibrium.■

10.2 Equilibrium with endogenous fixed cost

The proposition 4 is proven here. Suppose again that the firm 1 chooses the highest quality such that $q_1 \geq q_2$. First, the local equilibrium in qualities is determined for the hierarchy of attributes supposed. Second, it is showed that a perfect Nash equilibrium exists under Assumption 1.

At the last stage of the game, profits are given by the equations (2.8) and (2.9). We study the choice of attributes of *firm 1* :

$$\frac{\partial \Pi_1}{\partial q_1} = \frac{N}{\theta} u'(q_1) - F'(q_1) \tag{10.7}$$

At the equilibrium, the level of attributes chosen verifies $\frac{u'(q_1)}{\theta} = \frac{F'(q_1)}{N}$ and the second order condition is :

$$\left. \frac{\partial^2 \Pi_1}{\partial q_1^2} \right|_{\frac{\partial \Pi_1}{\partial q_1}=0} = \frac{N}{\theta} u''(q_1) - F''(q_1) \leq 0 \tag{10.8}$$

As utility is concave and fixed cost convex, profit is concave.

Consider now the quality choice of *firm 2*. After simplifications, the first derivative is :

$$\frac{\partial \Pi_2}{\partial q_2} = \frac{4Nu_1 x u'(q_2)}{\theta\sqrt{\Delta}(\sqrt{\Delta}+u_1)} - F'(q_2) \tag{10.9}$$

At this extreme, the level of attributes chosen by 2 verifies :

$$\frac{u'(q_2)}{\theta} \frac{4u_1 x}{\sqrt{\Delta}(\sqrt{\Delta}+u_1)} = \frac{F'(q_2)}{N}. \tag{10.10}$$

The second order condition is :

$$\left. \frac{\partial^2 \Pi_2}{\partial q_2^2} \right|_{\frac{\partial \Pi_2}{\partial q_2}=0} = \frac{4Nu_1[u_1 u'^2(q_2)(\sqrt{\Delta}+u_1+2x) + xu''(q_2)(u_1+\sqrt{\Delta})(u_1+4x)]}{\theta\sqrt{\Delta}(u_1+4x)(u_1+\sqrt{\Delta})^2} - F''(q_2) \leq 0 \tag{10.11}$$

The assumptions of convex fixed costs and concave utilities are not sufficient to ensure that profit is concave in the general case : the fixed cost function must also be sufficiently convex (or the utility strongly concave).

When profits are concave, the levels of attributes chosen verify $q_1 \geq q_2$ because cost and utility functions are identical between firms and moreover :

$$\frac{4u_1 x}{\sqrt{\Delta}(\sqrt{\Delta}+u_1)} = \frac{4u_1 x}{u_1^2 + 4u_1 x + u_1\sqrt{\Delta}} < 1 \tag{10.12}$$

Consequently, condition (2.4) holds and a local perfect Nash equilibrium exists.

Under Assumption 1 and when $\theta = N = 1$, levels of attributes at the equilibrium are $q_1^c = 0.5$ and $q_2^c = 0.317954$. The second order condition is always verified for firm 1 and the profit of firm 2 is strictly concave : $\left. \frac{\partial^2 \Pi_2}{\partial q_2^2} \right|_{\frac{\partial \Pi_2}{\partial q_2}=0} \approx -1.8$. The profits are given by $\Pi_1(q_1^c) \approx 0.25$ and $\Pi_2(q_2^c) \approx 0.28$.

The candidate highlighted here is really an equilibrium, as it is shown now. For the value of q_2^c computed, it can be easily shown by using equation (10.9) that the profit of *firm 1* is strictly increasing for all $q_1^{cc} \in [0, q_2^c]$. Thus, the optimal value when firm 1 deviates is $q_1^{cc} = q_2^c \approx 0.3179$. But $q_1^{cc} = q_2^c$ can not be an equilibrium because firm 1 has not chosen the value q_2^c during its choice of q_1^c inside the interval $[q_2^c; +\infty[$. Firm 1 thus prefers choosing $q_1^c = 0.5$ rather than deviating.

Finally, we have previously shown that *firm 2* chooses $q_2^{cc} = q_1^c$ if it deviates from q_2^c which implies that $q_2^{cc} = 0.5$. The profit obtained is $\Pi_2(q_2^{cc}) = \Pi_1(q_1^c) > \Pi_2(q_2^c)$ and thus the value $q_2^c = 0.3179$ is profit maximizing for the firm 2.

The shapes of the utility and costs functions in the Assumption 1 guarantee the existence of a perfect Nash equilibrium as no firm has an interest in modifying its choice at each stage.■